Design, print and test a race car made of composite shapes

Students will apply math principles to the design of a fun, recognizable object - a race car – that they can design, 3D print and race in order to test the relationship between volume, surface area and acceleration. They will use their race car model to observe what happens to the car’s acceleration when: 1) the shape of the car is changed while maintaining the same volume; and 2) when the shape of the car changes but the surface area does not.

Lesson from www.Tinkerine.com modified by Lisa M Suarez Caraballo
VOLUME + SURFACE AREA

[Math]

Overview
Students will apply math principles to the design of a fun, recognizable object - a race car – that they can design, 3D print and race in order to test the relationship between volume, surface area and acceleration. They will use their race car model to observe what happens to the car’s acceleration when: 1) the shape of the car is changed while maintaining the same volume; and 2) when the shape of the car changes but the surface area does not.

Teaching days required: 4
Note: this project is a capstone, or end of the year project. It can of course be used at any point during the school term if time permits.

• Day 1: Teach/review volume and surface area, students working in groups design their car and model it in a 3D modelling program, prepare the files for 3D printing during class time, begin 3D printing and continue outside of class time to have prints ready for the next class
• Day 2: Students test their models and record data
• Day 3: Students redesign their models based on their test results and 3D print new versions
• Day 4: Students test their new models and record data

Rationale: Why incorporate 3D Printing (3DP)?
When teaching students the concept of surface area and volume, students are generally given tasks to calculate the volume and surface area of composite shapes on paper. This kind of lesson is isolated from real world applications and thus math is viewed as something that is disconnected from the real world. Some teachers have students make model pencil cases and model cars using composite shapes. Students generally use materials they find at home, such as paper and cans, to make these objects. The result is these objects are imprecise and difficult to modify. And due to their fragile nature, these homemade projects tend to be static – that is, they cannot actually be used or tested under real conditions.

By using 3D modelling software to combine composite shapes and a 3D printer to print a physical representation of the model, cars can be created with greater precision than by building objects using household materials. Modifications to these objects can also be made faster: a student simply enters the new dimensions into the modelling software and immediately afterwards prints the object. Because 3D printed objects are made with a hard plastic, they objects can be tested under real conditions, and different versions of the same object can be tested to see which version produces optimal results.

Materials required for teaching this lesson
• 3D printer
• Computer
• Tinkercad: use for free on a computer browser at https://tinkercad.com
• Ruler, pen, paper, eraser, calculator
Prerequisite Knowledge

- 2-dimensional (2D) object: a shape with only 2 dimensions, (e.g. only length and width, no height/thickness)
- 3-dimensional (3D) object: a shape with only 3 dimensions, (e.g. with length, width and height/thickness)
- Area: the size of a surface; the amount of space inside the boundary of a 2-dimensional object
- Composite Shape: a figure made up of two or more geometrical shapes
- Surface area is the sum of all the areas of all the shapes that cover the surface of the object.

The following formulas allow you to determine the surface area of the specified three-dimensional shape:

- Surface area of a rectangular prism = 2(width)(height) + 2(length)(height) + 2(length)(width)
- Surface area of a right pyramid = \(\text{[base area]} + \cdots \times \text{Perimeter} \times \text{[Slant Length]}\)
- Surface area of a right cylinder = \(2\pi r^2 + 2\pi rh\)
- Surface area of a right cone = \(\pi r(r + \sqrt{h^2 + r^2})\)
- Surface area of a sphere = \(4\pi r^2\)

Volume is the amount of three-dimensional space an object occupies. The following formulas allow you to determine the volume of the specified three-dimensional shape:

- Volume of a rectangular prism = width \(\times\) length \(\times\) height
- Volume of a right pyramid = \(\frac{1}{3} \times \text{[base area]} \times \text{height}\)
- Volume of a right cylinder = \(\pi r^2 \times \text{height}\)
- Volume of a right cone = \(\frac{1}{3} \times \pi \times r^2 \times \text{height}\)
- Volume of a sphere = \(\frac{4}{3} \times \pi \times r^3\)

Learning Outcomes

Math 8

- determine the surface area of right rectangular prism, right triangular prism, right cylinders to solve problems
- develop and apply formulas for determining the volume of right prisms and right cylinders

Math 9

- determine the surface area of right rectangular prism, right triangular prism, right cylinders to solve problems
- determine the surface area of composite 3-D objects to solve problems

Foundations and Pre-Calculus Math 10

- Solve problems, using SI and imperial units that involve the surface area and volume of 3D objects, including: right cones, right cylinders, right prisms, right pyramids and spheres
What YOU will be making with 3DP

A sample 3D printed “race car” made of calculated volumes.

Overview of TinkerCad software
Instructions for Making
Model a race car in Tinkercad – **Use the Data Collection sheet as you create the race car to keep track of its dimensions.**

1. Login to Tinkercad.com. Select “Create New Design”.
2. Under the “Geometric” heading on the right menu, click and drag a “Box” onto the workplane.

3. Under the “Helpers” heading on the right menu, click and drag a “Ruler” onto the workplane. Place ruler on the lower left corner of the box.

4. Click the Depth dimension. Enter a value of 10mm. Click to Height dimension. Enter a value of 16mm.

Which dimension represents the depth? Height?
5 Click “X” to Dismiss ruler.

6 Repeat steps 2-5. This time enter a depth of 22mm and a height of 16mm.

7 Click and drag one box to touch the other box. Make sure the bottom edges of each box are aligned.

8 Hold down the Shift key and select both boxes. Select the Group button on the top menu to join objects together.
9  Repeat steps 2-5. This time enter a depth of 12mm and a height of 16mm.

10 Repeat steps 7-8.

11 Under the “Geometric” heading on the right menu, click and drag a “Cylinder” onto the workplane.

12 Click and drag a ruler onto the workplane and place on the lower left corner of the cylinder. Enter a width and depth of 16mm.
13 Change your view so that you are looking straight down onto the figures. Line up the cylinder so that it is aligned at the center of the 1st box we dragged onto the workplane. Move the cylinder so that it touches the base of the 1st box.

14 Click and drag the cylinder so that it moves 5mm up into the 1st box. You will notice the measurement appear as soon as you start moving the cylinder up into the rectangular prism.

15 Repeat steps 11-14 but for the 3rd box we dragged onto the workplane.

16 Hold the Shift key and select both cylinders. Select the Hole box in the Inspector Menu.
17 Hold the Shift key and select the cylinders and the boxes. Select Group on the top menu.

18 To create the wheels, repeat step 12, place a cylinder on the workplane, but enter a width and depth of 16mm and a height of 45mm.

19 Repeat step 12 but enter a width and depth of 17mm and a height of 5mm.
20 Hold Shift and select both cylinders. Select the Adjust icon on the top menu and the Align feature.

![](image1.png)

21 Click the center dot to align the centers of the cylinders.

![](image2.png)

22 Click the dot to align the cylinders on top of each other. Dismiss tool after.

![](image3.png)

23 Click to select the smaller cylinder. Click the black cone icon that appears above the cylinder and drag the cylinder up 9mm.

![](image4.png)
Hold the Shift key and select both cylinders. Press Group.

Repeat steps 19-21. Drag the shorter cylinder 31mm up the longer one. Select both cylinders when finished and Group.

Select the cylinder, go to the Edit tab on the top menu, and select Duplicate. Click the cylinder and drag. You’ll find that there are now two identical cylinders. Position the 2nd cylinder away from the first. You’re finished!

3D Print the race car

Race car component parts dimensions:
Body: 60mm (w) x 16mm (d) x 22mm (h)
Wheels and Axle: 17mm (diameter) x 45mm (h)
**VOLUME + SURFACE AREA: STUDENT WORKSHEET**

**Introduction:** You have been selected to participate in the Math Car Derby Race. In order to compete, you must design and 3D print the vehicle you will enter into the race. The vehicle design is to be comprised of combined shapes to form a 3D composite figure. The race vehicle must be of certain dimensions. Cars will be raced on a ramp and the winner will be the car that travels the longest distance before coming to rest. Cars will participate in a preliminary round race first. After the preliminary round, modifications will be allowed to the vehicles before racing in the final round.

You will test the relationship between volume, surface area and acceleration. You will use your race car model to observe what happens to the car’s acceleration when: 1) the shape of the car is changed while maintaining the same volume; and 2) when the shape of the car changes but the surface area does not.

**Purpose:** Students will be able to solve problems, using SI and imperial units that involve the surface area and volume of 3-D objects, including: right cones, right cylinders, right prisms, right pyramids and spheres.

**Materials**
- 3D printer
- Computer
- Tinkercad: use for free on a computer browser at [https://tinkercad.com](https://tinkercad.com)
- Ruler, pen, paper, eraser, calculator
- Instructions for Making

**Procedure**

1. On a sheet of paper, design your 3-D composite vehicle to use in the car derby in Tinkercad. This vehicle must satisfy the design restrictions below involving length, surface area and volume of the object. This composite object is the vehicle that will be used in the preliminary car derby race:
   A. Your vehicle design must be a 3-D composite figure made up of at least 3 different shapes studied in our math unit. These include: rectangular prism, triangular prisms, cylinders, cones, pyramids and spheres.
   B. The total volume of your 3-D vehicle must not exceed 350 cm³ (may need to alter depending on 3-d printer capability)
   C. The total surface area of your 3-D vehicle must not exceed 400 cm² (may need to alter depending on 3-d printer capability)
   D. The length of your 3-D vehicle must not exceed 12 cm (may need to alter depending on 3-d printer capability)
   E. The width of your 3-D vehicle must not exceed 8 cm (may need to alter depending on 3-d printer capability)
   F. The height of your 3-D vehicle must not exceed 5 cm (may need to alter depending on 3-d printer capability)

You may also include other shapes in your design, but be aware it will be difficult for you to calculate the surface area and volume of those shapes. All parts of your vehicle must be accounted for in the volume and surface area calculations.

2. Complete Table 1 under Data.
3. Login to Tinkercad.com and create your 3-D vehicle using your sketch as the design plan. Refer to the instructions for making as an example for designing and printing your vehicle.
4. Test your car in the preliminary race. Ensure that you are taking notes on how your car performs and how your competitor’s cars are performing. Come up with ideas for improving your car design. These notes will aid in your re-design before the final race.
After participating in the preliminary race and observing how your car performed, go back and make modifications to your car design to determine if your car can travel an even greater distance. You are allowed to make modifications to the types of shapes used and the size of the shapes. However, you must ensure your new modifications meet the design restrictions for the race, or else your car will be disqualified.

Complete Table 2 under Data.

3-D print your re-designed vehicle

Use your car in the race to see if you have designed the car that travels the greatest distance.

When handing in your project for evaluation, make sure to also include the following:

A. Rationale. Submit a write up of a maximum of 100 words describing why you chose specific shapes to use in your 3D vehicle design and how you decided on the various measurements of each object.

B. Images. You are required to photograph a picture of your 3D composite vehicle. In addition to a general photo, take a picture of the front view, right side view and top view.
Data:
Complete the following table listing the measurements and calculation of surface area and volume of the objects that make up your 3-D composite vehicle. You can copy and paste as many rows as needed in order to do the calculations for all shapes involved.

Part 1: Data from 3-D vehicle used for the Preliminary Round of the race

<table>
<thead>
<tr>
<th>Description of car part</th>
<th>Shaped used</th>
<th>Key Measurements</th>
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<tbody>
<tr>
<td>E.g. wheel</td>
<td>Cylinder</td>
<td>Radius = 1 cm</td>
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<tr>
<td></td>
<td></td>
<td>Thickness = 0.8 cm</td>
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<tr>
<td></td>
<td></td>
<td>Surface Area = 11.3 cm²</td>
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<tr>
<td></td>
<td></td>
<td>Volume = 2.5 cm³</td>
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</tbody>
</table>


Part 2: Data from MODIFIED parts of 3-D vehicle used for the Final Round of the race

*** NOTE: Use this sheet to modify the race car using your own design BUT make sure you keep the volume and surface area the same. Clearly state which parts of the vehicle have been modified.

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Attach supporting calculations for surface area and volume below.

Analysis
Prior to the Preliminary Race: Submit a write up of a maximum of 100 words describing why you chose those specific shapes to use in your 3-D vehicle design and how you decided on the various measurements of each part.

After the Preliminary Race and Before the Final Race: Submit a write up of a maximum of 100 words describing why you chose to make the specific modifications to your 3-D vehicle. If you did not need to make any modifications, please justify why you chose not to make any changes.

After the Final Race: Submit a write up of a maximum of 100 words describing why you chose to make the specific modifications to your 3-D vehicle. If you did not need to make any modifications, please justify why you chose not to make any changes.